

Half Yearly Examination - 2018-19

Mathematics

Class : XI

Time : 3 Hrs. + 15 min.

Full Marks : 100

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time)

The Question Paper consists of three Sections A, B and C

Candidate are required to attempt all questions from Section A and all questions

EITHER from Section B OR Section C

(All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets. [ ].

Section A (80 Marks)

- 1 i) Find the domain and range of the function  $f(x) = \frac{|x-3|}{x-3}$  [2]
- ii) Show that  $A \cap B = A \cap C$  need not imply  $B=C$  [2]
- iii) Express in the standard form  $(-2 + 5i)^3$  [2]
- iv) Find the value of  $a$  if one root of the equation  $8x^2 - 6x + a = 0$  is the double the other [2]
- v) Prove that  $\cos 5^\circ - \sin 25^\circ = \sin 35^\circ$  [2]
- vi) If  $\cos x = \frac{-2}{5}$  find  $\cos 3x$  [2]
- vii) Find principal solution of  $\sec x = -2$  [2]
- viii) Find the sum of  $3+7+11+15+\dots$  to 30 terms [2]
- ix) Prove by mathematical induction  $3x+6x+9x+\dots$  to  $n$  terms  $= \frac{3}{2}n(n+1)x$  [2]
- x) Find the angle between the lines  $2x-5y=3$  and  $2x=6y-2$  [2]
- 2 Find the sum of the series,  $\left(1 + \frac{1}{2^2}\right) + \left(\frac{1}{2} + \frac{1}{2^4}\right) + \left(\frac{1}{2^2} + \frac{1}{2^6}\right) + \dots$  [4]

If  $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$  Prove that  $\frac{\tan x}{\tan y} = \frac{a}{b}$   
 Or

If  $\cos(x+2y) = m \cos x$ , prove that  $\cot y = \frac{1+m}{1-m} \tan(x+y)$

4. If  $x = a + b\omega + c\omega^2, y = a\omega^2 + b + c\omega, z = a\omega + b\omega^2 + c$  then find the value of  $\frac{z^2}{yx} + \frac{y^2}{zx} + \frac{x^2}{yz}$  [4]

5. Show that the function  $f; N \rightarrow N$  defined by  $f(x) = 3x - 1$  is one one but not on to. [4]

6. Prove by mathematical induction  $3^{2n+2} - 8n - 9$  is divisible by 64 [4]

7. Find the points on the line  $y=x$  which are at a distance of 5 units from the line  $4x+3y-1=0$  [4]  
 OR

Show that the path of a moving point such that its distance from 2 lines  $3x-2y=5$  and  $3x+2y=5$  are equal is a line.

8. A triangle is formed by the lines  $3x+4y=10, 4x-3y=5$  and  $7x+y+10=0$  find the internal bisector of the angle opposite to the side  $7x+y+10=0$  [4]

9. i). find the square root of  $7-24i$  [4]

ii) Convert in to polar forms.  $\frac{1+3i}{1-i}$

10. If  $x, y, z$  are in AP prove that  $\cot y = \frac{\sin x - \sin z}{\cos z - \cos x}$  [4]

11. Find the sum of  $1 + 2.2 + 3.2^2 + 4.2^3 + \dots + 100.2^{99}$  [6]

12. Find the sum of  $5+7+13+31+85 + \dots$  Up to  $n$  terms [6]

13. If  $\tan y = 3 \tan x$ , prove that  $\tan(x+y) = \frac{2\sin 2y}{1+\cos 2y}$  [6]

14. If one root of the quadratic equation  $ax^2 + bx + c = 0$  is equal to the  $n$ th power of the other root, [6]

then show that  $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$

**Section B**

(Only for XI A and B students)

15. a) Find the points on the x axis at a distance of  $5\sqrt{2}$  units from the point  $(2, -3, 6)$ . [2\*4]  
 b) In what ratio is the line segment joining the points A  $(-2, 4, 7)$  and B  $(3, -5, 8)$  divided by the XZ plane? Also find the coordinates of the point of division.  
 c) Find the coordinates of the point which divides the join of points  $(2, -1, 3)$  and  $(-4, 3, 2)$  and  $(3, 5, -7)$  and  $(3, -8, 9)$   
 d) Find the centroid of a triangle, the midpoints of whose sides are  $(1, 2, -3), (3, 0, -6)$  and  $(2, 3, -8)$  [4]

16. Using section formula, prove that the points  $(-4, 6, 10), (2, 4, 6)$  and  $(14, 0, -2)$  are collinear. [4]

17. If the origin is the centroid of the triangle PQR with vertices P  $(2a, 2, 6), (-4, 3b, -10)$  and R  $(8, 14, 2c)$  then find the values of  $a, b, c$ . [4]

18. Find the angle between the line segment joining the points  $(2, 3, -6)$  and  $(-9, 9, 2)$  and  $(3, -5, -3)$  and  $(-5, 6, -5)$

**Section C**

(Only for XI C and D students)

19. Two sample sizes of 50 and 100 are given. The mean of these samples respectively are 56 and 50 and standard deviation of these are 4 and 5. Find the mean and standard deviation of size 150 by combining the two samples [6]

20. Find the mean, median and mode for the data given below [6]

Class	11-15	15-19	19-23	23-27	27-31	31-35	35-39
Freq.	15	25	25	24	15	14	12

21. Find the quartile deviation and mean deviation about median of the following data given [8]

Class	11-15	15-19	19-23	23-27	27-31	31-35	35-39
Freq.	18	21	22	28	15	14	12