Quarterly Examination - 2018-19
MATHEMATICS

Class : XII

Time : 3 Hrs. 15 mints **Full Marks : 100**

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1 i Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}$ 2

ii If $\begin{bmatrix} x+y \\ x-y \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ then find the value of x and y 2

iii $\int x\sec^2 x\,dx$ 2

iv Find the principal value of $\tan^{-1}1+\cos^{-1}\frac{-1}{2}$ 2

v Show that the function f defined as follows is continuous at x=2 $f(x)=\begin{cases} x-2, & 0\le x\le 1 \\ 2x^2-x, & 1< x\le 2 \\ 5x-4, & 2< x \end{cases}$ 2

vi Evaluate : $\int\frac{dx}{x-\sqrt{x}}$. 2

vii Evaluate $\int\frac{x+1dx}{x(x+\log x)}$ 2

viii Without expanding evaluate $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ 2

ix Differentiate $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ 2

x If $A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2-4A+3I=0$ .Hence find A^{-1} 2

2 Prove by using properties that 4

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ a+b & b+c & c+a \end{vmatrix}=a^3+b^3+c^3-3abc$$

3 Solve by Martine's rule x +2y+z = 1, 2x+3y +2z=1.3x+y-2z=0 4

4 If $\sin^{-1}x+\tan^{-1}x=\frac{\pi}{2}$ Prove that $2x^2+1=\sqrt{5}$ 4

5 Solve $\sin^{-1}(1-x)-2\sin^{-1}(x)=\frac{\pi}{2}$ 4

6 Prove that 4

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

7 Differentiate $\tan^{-1}\frac{\left(\sqrt{1+x^2}\right)-1}{x}$ 4

8 Find $\frac{dy}{dx}$, if $x^{\sin x}+(\sin x)^x=a^2$ 4

9 Evaluate a) $\int\frac{x+1dx}{\sqrt{2x^2+3x+5}}$

10 Evaluate a) $\int\tan^{-1}\sqrt{x}\,dx$, 4

11 Find the product of matrices A and B ,. 6

$$A=\begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \text{ and } \mathbf{B}=\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix},$$

and hence solve the following equations; x-2y+3z=6 , x+4y+z=12, x-3y+2z=1

12 Find the inverse of the matrix $A=\begin{bmatrix} 1 & -2 & 3 \\ 1 & -4 & 1 \\ 1 & -3 & -2 \end{bmatrix}$ using elementary transformations 6

13 Evaluate $\int\sqrt{\cot x}.dx$ 6

14 $\int\left(\log(\log x)+\left(\frac{1}{\log x}\right)^2\right)dx$ 6

Section B

(Only for Section A and B Students)

15 If the coordinates of 4 points are $A(2,3,4), B(5,4,-1), C(3,6,2), and D(1,2,0)$ then show that AB is perpendicular to CD. 4

16 Find the area of a parallelogram whose diagonals are $6\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{k}$. 4

17 Find k so that the four points are coplanar $A(-1,4,-3), B(3,k,-5), C(-3,8,-5), and D(-3,2,1)$ 4

18 If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that $\vec{b}+\vec{c}, \vec{c}+\vec{a}, \vec{a}+\vec{b}$ are coplanar 4

19 If $\vec{a}$ and $\vec{b}$ are unit vectors and is the angle between them. . Then prove that $\frac{\theta}{2} = \frac{1}{2}\left|\vec{a}+\vec{b}\right|$ 4

Section C

(Only for commerce students)

20 i. Find the marginal cost and average cost of the cost function $C(x)=\frac{x^4}{3}+3x^2-7x+\frac{16}{x}$. 4

ii. Find the profit function and hence find the breakeven points when p=72-4x, $C(x)=16x+130$

21 Prove that the slope of the average cost curve is $\frac{1}{x}(MC-AC)$ for the total cost function $C(x)=\frac{x^3}{3}+3x^2-7x+16$ 4

22 The demand function of a monopolist is $p(x)=300-x^2$, find the revenue function, the marginal revenue function, and the out put x when revenue is maximum 6

23 If $C(x)=2x\left(\frac{x+4}{x+1}\right)+6$ show that marginal cost falls continuously as x increases. 6