Quarterly Examination 2017-2018

Std. : XII | **Full Marks : 100**

Subject : MATHEMATICS | **Time : 3Hrs+15Min.**

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1. i. Solve by crammer's rule x+2y+z = 4, 2x+3y − 2z = 3.3x+y+z = 5 [2]

ii. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ then find the value of x so that $A^2 = xA - 2I$ [2]

iii. Find $\frac{dy}{dx}$ if y = $\operatorname{cosec}^{-1} \frac{1+x^2}{2x}$ [2]

iv. Solve $\cos(2Sin^{-1}x) = \frac{1}{9}$ [2]

v. Evaluate $\tan^{-1}$ [2 cos(2 $\sin^{-1} \frac{1}{2}$] [2]

vi. Evaluate : $\int \frac{(x+1)\,dx}{\sqrt{x^2+4x+1}}$ [2]

vii. Evaluate : $\int \frac{x - 1dx}{X^2+2x-1}$ [2]

viii. Prove that $(x + y)^I = x^I + y^I$ [2]

ix. Differentiate $\frac{1-x^2}{1+x^2}$ [2]

x. evaluate $\int \frac{dx}{1+3\sin^2 x}$ [2]

2. Prove by using properties that [4]

$$\begin{vmatrix} \sin^2 x & \sin x & \cos^2 x \\ \sin^2 y & \sin y & \cos^2 y \\ \sin^2 z & \sin z & \cos^2 z \end{vmatrix} = -(\sin x - \sin y)(\sin y - \sin z)(\sin z - \sin x)$$

3. Solve by Martine's rule $x + 2y + z = 4$, $2x + 3y - 2z = 3.3x + y + 2z = 6$ [4]

4. Prove that $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. [4]

5. Solve $\tan^{-1}(x+1) + \tan^{-1}(x-1) + \tan^{-1} x = \tan^{-1} 3x$ [4]

6. Prove that $\begin{vmatrix} 1 & z & z^2 - xy \\ 1 & x & x^2 - yz \\ 1 & y & y^2 - zx \end{vmatrix} = 0$ [4]

7. Find if $y\sqrt{x^2 + 1} = \log\left(x + \sqrt{x^2 + 1}\right)$ prove that $(x^2 + 1)\dfrac{dy}{dx} + xy = 1$ [4]

8. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the equations,
$-x + y + z = 0,\ x - 3y + z = 2,\ x + 2y + z = 4,\ -$ [4]

9. Prove that $\begin{vmatrix} a+b & b+c & c \\ b+c & c+a & a \\ c+a & a+b & b \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)$ [4]

10. i. Evaluate a) $\int \dfrac{dx}{2x^2 + 3x + 5}$ b) $\int \dfrac{sec^2\, xdx}{\sqrt{tan^2\, x + 4}}$ [6]

11. i. If $x = \sin t$, $y = \sin pt$, prove that $(1 - x^2)\, y_2 - xy_1 + p^2\, y = 0$ [3]

ii If $x = a(l+\cos t)$, and $y = a\,(t + \sin t)$ find y_2 at $t = \dfrac{\pi}{2}$ [3]

12. i. a) Evaluate $\int \dfrac{sec2\,(2tan^{-1}\, x)}{1+x^2}\, dx$ b) $\int \dfrac{x + 1}{x^2 + 4x + 5}\, dx$ [6]

13. ii. Solve $\sin^{-1}x + \sin^{-1}\, y = \dfrac{2\pi}{3}$ *and* $\cos^{-1}\, x - \cos^{-1}\, y = \dfrac{\pi}{3} =$ [6]

Section B

(Only for Section A and B Students)

14. Show that the four points with position vectors [4]
$-\hat{i} + 4\hat{j} - 3\hat{k},\ 3\hat{i} + 2\hat{j} - 5\hat{k},\ -3\hat{z} + 8\hat{j} - 5\hat{k}$ and $-3\hat{i} + 2\hat{j} + \hat{k}$ are coplanar.

15. Find the area of a triangle formed by the points whose position [4]
vectors are $3\hat{i} + \hat{j},\ 5\hat{i} + 2\hat{j} + \hat{k},\ \hat{i} - 2\hat{j} + 3\hat{k}$.

16. Find the unit vector perpendicular to each of the vectors $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{k}$. [4]

17. If $|a| = 3$, $|b| = 4$, $|c| = 5$ and a,b,c are respectively perpendicular to b+c, c+a, a+b find the value of $|a + b + c|$. [4]

18. If $\vec{a}$ and $\vec{b}$ are unit vectors and is the angle between them. Then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ [4]

Section C

(Only for Section D and E students)

19. The demand function for a manufacturer's product is $p = 20 - \frac{x}{4}$ where x is the number of units and p is the price per unit. At what value of x will there be maximum revenue ? What is the maximum revenue ? [4]

20. Prove that the slope the average cost curve is $\frac{1}{x}$ (MC − AC) for the total cost function $C(x) = ax^3 + bx^2 + cx + d$. [4]

21. The fixed cost of a new product is Rs. 35,000 and the variable cost per unit is Rs. 500. If the demand function is p(x) = 500 − 100 x, find: i) The profit function ii) break even values iii) the values of x that result in loss. [4]

22. The total cost and demand function of an item are given by $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and p = 100 — x respectively Write the total revenue function and the profit function. Find the number of items when the profit will be maximum. [4]

23. The average cost function AC for a commodity is given by AC=x+5+36/x in terms of output x, find the

i) total cost and marginal cost as the function of x

ii) output for which AC increases.

iii) Show that the marginal average cost is given by $\{x\, MC - C(x)\} / x^2$?