FIRST PRE-BOARD EXAMINATION 2020-2021

MATHEMATICS

(Time allowed: Three hours)

Class: 12
Marks:80

(Candidates are allowed additional 15 minutes for only reading the paper.
They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions **from Section A and all questions***
EITHER from Section B OR Section C

***Section A:** Internal choice has been provided in two **questions of four marks each and two** questions of six marks each.*
***Section B:** Internal choice has been provided in one question of 2 marks and one question of four marks.*

***Section C:** Internal choice has been provided in one question of 2 marks and one question of four marks*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION A (65 Marks)

[15×2]

Question 1

i). A homogeneous differential equation has infinitely many solution. Justify.

ii). Solve the differential equation $dy - x\,dx = 0$, if the curve passes through (1, 0)?

iii) The values of k which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} + \frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ______

iv). State whether the following condition is true or false?
"In Bayesian theorem, it is important to find the probability of both the events occurring simultaneously."
a). true b). false

v). Integration of function $y = f(x)$ from limit $x_1 < x < x_2$, $y_1 < y < y_2$, gives ___________

a) Area of f(x) within $x_1 < x < x_2$
b) Volume of f(x) within $x_1 < x < x_2$
c) Slope of f(x) within $x_1 < x < x_2$
d) Maximum value of f(x) within $x_1 < x < x_2$

vi). A differential equation is considered to be ordinary if it has

a). one dependent variable

b). more than one dependent variable

c). one independent variable

d). more than one independent variable

vii). On which of the mentioned points is the Bayesian theorem reasonable to apply?

a). independent events

b). dependent events

c). neither a or b

d). either a or b

viii). If $dy = x^2\,dx$; what is the equation of y in terms of x if the curve passes through (1, 1).

ix). Vector has

a). direction

b). None of these

c). magnitude

d). magnitude as well as direction.

x) Naina receives emails that consist of 18% spam of those emails. The spam filter is 93% reliable i.e., 93% of the mails it marks as spam are actually a spam and 93% of spam mails are correctly labeled as spam. If a mail marked spam by her spam filter, determine the probability that it is really spam.

a) 50%

b) 84%

c) 39%

d) 63%

xi). Physically, integrating $\int_a^b f(x)dx$ means finding the

a). area under the curve from a to b

b). area to the left of point a

c). area to the right of point b

d). area above the curve from a to b

xii). Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2, respectively, having $\vec{a}.\vec{b} = \sqrt{6}$

a) $\frac{\pi}{2}$ b). $\frac{\pi}{4}$ c). $\frac{\pi}{3}$ d). $\frac{\pi}{5}$

xiii). Integration of (Sin(x) + Cos(x)) e^x is ______________

a) e^x Cos(x)

b) e^x Sin(x)

c) e^x Tan(x)

d) e^x (Sin(x)+Cos(x))

xiv). Vectors are not associative under dot product whereas if they are perpendicular to each other, then associative Justify.

xv) Among which of the following mentioned statements can the Bayesian probability be applied?

a) In the cases, where we have one event

b) In the cases, where we have two events

c) In the cases, where we have three events

d) In the cases, where we have more than three events

Question 2

Evaluate $\int \frac{1}{1+\cos x} dx$ [2]

OR

Evaluate $\int \frac{1}{x^3+1} dx$

Question 3

Prove that if E and F are independent events, then the events E and F^c are also independent [2]

Question 4

Solve the differential equations [2]

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

OR

Find the particular solution of the differential equation

$$(1 - y^2)(1 + \log x)dx + 2xy\, dy = 0$$

Question 5

Prove that $P(B) = P(B|A) \times P(A) + P(B|A^c) \times P(A^c)$ [2]

Question 6

Show that $y^2 = 4a(x + a)$ is a solution of the differential equation $y(1 - {y_1}^2) = 2xy_1$ [2]

Question 7

$\int \frac{\sqrt{2}\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx = ax + b \log\left|\sin\left(x - \frac{\pi}{4}\right) + c\right|$, find the value of a and b. [4]

Question 8

Find the direction cosines of the sides of the triangle whose vertices are (3, 5, - 4), (- 1, 1, 2) and (- 5, - 5, - 2). [4]

Question 9

Show that the line through the points $(4, 7, 8), (2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1), (1, 2, 5)$. [4]

OR

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ are perpendicular to each other

Question 10

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six [4]

OR

Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

Question 11

Find the solution of the differential equation $\frac{dv}{dt} = 9.8 - 0.196v$

[6]

OR

a) Form the equation of a curve passing through the point (0,0) and whose differential equation is $\frac{dy}{dx} = e^t sinx$

b) Form the differential equation representing the family of curve $y = mx$, where m is an arbitrary constant.

Question 12 **[6]**

a). What are the direction cosines of a line which makes equal angles with the coordinate axes?

b). If two vectors are perpendicular to each other in plane then prove that magnitude of their cross product is a scalar quantity.

c). Find the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q(4, 1, −2)

Question 13

Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold? **[6]**

OR

A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from

(a) machine A (b) machine B (c) machine C?

Question 14

a). Find sine of the angle between the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$.

b). If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$ **[6]**

SECTION B (15 Marks)

Question 15 **[5 x 1=5]**

i). what is the probability that a leap year has 53 Sundays?

ii). Define magnitude of cross product and give an example.

iii). Let A and B are two events associated with a sample space Ω. Prove that $P(A \cup B)$ lies between 0 and 1.

iv). Every independent events are mutually exclusive. Justify with example.

v). Let L_1 be a line passing through the point $A(x_1, y_1, z_1)$ and parallel to Y-axis, then find vector equation.

Question 16

Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$. $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, then check whether $\vec{c}$ is **perpendicular to both $\vec{a}$ and $\vec{b}$** [2]

Question 17

Solve $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$ [4]

OR

Show that the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

Question 18

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point. **[4]**

SECTION C (15 Marks)

Question 19 **[5×1]**

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v), answer the questions as instructed.

(i) The demand function of a monopolist is given by $x = 100 - 4p$. The quantity at which MR = 0 will be:

(a) 25 (b) 10 (c) 50 (d) 30

(ii) If the lines of regression are parallel to coordinate axes, then the coefficient of correlation is:

(a) 1 (b) 0 (c) −1 (d) ½

(iii) Find the marginal cost function (MC), if the cost function is : $C(x) = x\ 3\ 3 + 5x\ 2 - 16x +$
2.

(iv) If revenue function $R(x) = 3x\ 3 + 8x - 2$, find the average revenue function.

(v) If $\sigma x = 3$, $\sigma y = 4$ and $bxy = 1\ 3$, then find the value of correlation coefficient (

Question 20 **[2]**

(a) A company produces a commodity with rupees 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of rupees 8 per unit. Find the break-even point.

OR

(b) The total cost function for a production is given by $\frac{3}{4}x^2 - 7x + 24$
Find the number of units produced for which M.C. = A.C.
(M.C.= Marginal Cost and A.C. = Average Cost.)

Question 21 [4]

Find the equation of the regression line of y on x, if the observations (x, y) are as follows:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Also, find the estimated value of y when x = 14.

Question 22 [4]

(a) A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood
Which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for rupees 48 per unit and product B is sold for rupees 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income? Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.

OR

(b) A farm is engaged in breeding hens. The hens are fed products A and B grown in the farm which contains nutrients P, Q and R. One kilogram of product A contains 36 units, 3 units and 20 units of nutrients P, Q and R respectively, whereas one kilogram of product B contains 6 units, 12 units and 10 units of nutrients P, Q and R respectively. The minimum requirement of nutrients P, Q and R for a hen is 108, 36 and 100 units respectively. Product A costs rupees 20 per kilogram and product B costs rupees 40 per kilogram. Using linear programming, find the number of kilograms of products A and B to be produced to minimize the total cost. Identify the feasible region from the rough sketch.