PRE ISC EXAMINATION 2017-18

CLASS: XII
Time: 3 Hrs.

MATHEMATICS

F.M.:100

(Candidates are allowed additional 15 minutes for only reading the paper,
They must NOT start writing during this time.)

The Question Paper consists of **three** sections A, B and C.

Candidate are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C**

Section A: Internal choice has been provided in three questions of four marks each and two questions of six marks each.

Section B: Internal choice has been provided in two questions of four marks each.

Section C: Internal choice has been provided in two questions of four marks each.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer. The intended marks for questions or parts of questions are given in brackets. [ ]

Section A

1 a) A binary operation * defined on Q-{1} is given by a * b = a+b – ab. Show that * is commutative as well as associative [2]

b) Without expanding at any stage, find the value of the determinant [2]

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

c) If $A=\begin{bmatrix} 0 & y \\ 4 & x \end{bmatrix}$ is skew symmetric matrix find the values of x and y [2]

d) Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x\, dx.$ [2]

e Find $\frac{dy}{dx}$ if $x = a(1+\sin t)$ and $y = a\cos t$ [2]

f Find the differential equation of the family of circles, with center(1,2) [2]

g Evaluate $\lim_{x \to 0} \frac{x^3 \cot x}{1-\cos x}$ [2]

h A card is drawn from a well shuffled pack of playing cards. What is the probability that it is neither a spade nor a King [2]

i Two dice are thrown simultaneously. What is the probability of obtaining a total score of 7? [2]

j Prove that: $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{77}{85}\right)$. [2]

2 Let A : R-{3} and B : R-{1}. Consider the function $F: A \to B$ *defined by* $f(x) = \frac{x-2}{x-3}$. Show that inverse of F exists and find F^{-1} [4]

[4]

3a Show that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$.

OR

b If x, y, z are different and $A = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$. [4]

4 Solve: $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) - \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \sin^{-1}\left(\frac{1+x}{1+x^2}\right)$ [4]

5a Show that the function $f(x) = x^2$ is differentiable at x=1 and hence find $f'(1)$ [4]

OR

b Using Langrage's mean value theorem , find a point on the curve $y = \sqrt{x-2}$ defined in the interval $[2,3]$ where the tangent is parallel to the chord joining the end point of the curve [4]

6 If $y = (\tan^{-1} x)^2$ prove that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$ [4]

7a $\int_1^2 \frac{dx}{(x+1)(x^2-7x+12)}$.

OR

b Evaluate as a limit of a sum $\int_2^4 (\quad x^3 + 2x + 1 \quad) dx$ [4]

8a Water is leaking from a conical funnel at the rate of $5cm^3$ / sec. if the radius of the base of the funnel is 10 cm and its height is 20cm, find the rate at which the water level is dropping when it is 5 cm from the top.

OR

b Find the equation of the tangent to the curve $3x^2 - y^2 = 2$ which are perpendicular to line x+3y=2. [4]

9 Solve: $(x^2+1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$, given that y(0)=0. [4]

10 A company has two plants to manufacture scooters. Plant I manufactures 60% of the scooters and Plant II manufactures 40%. At plant I, 75% of scooters are rated as of standard quality and at Plant II, 80% of scooters are rated as of standard quality. A scooter is chosen at random and is found to be of standard quality. What is the probability that it has come from Plant II? [6]

11a Show that that following equations are consistent $x - 2y + z = 0$, $y - z = 2$, $2x - 3z = 10$.
Also find the solution using matrix method.

OR

b Using elementary transformations, find the inverse of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 10 \\ 9 & 8 & 7 \end{bmatrix}$ [6]

12a The lengths of three sides of a trapezium are equal, each being 10 cm. Find the maximum area of such a trapezium.

OR

b Show that the volume of the largest circular cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.

13 Evaluate $\int \frac{dx}{(x^4+1)}$ [6]

14a Three persons A, B, C, shoot to hit a target. If A hits the target 4 times in 5 trials, B hits it in 3 times in 4 trials and C hits it in 2 times in 3 trials. Find the probability that (i) exactly two persons hit the target (ii) at lest two persons hit target. [6]

b The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9, if the probability of getting at least one contract is 4/5 what is the probability that he will get both?

SECTION B (20 Marks)

15a Find the area of the triangle whose adjacent sides are given by the vectors $2\vec{i}-\vec{j}+2\vec{k}$ and $3\vec{i}-2\vec{j}+\vec{k}$ [2]

b [2]

c Find the angle between the planes $2x+3y+4z=5$ and $3x+4y+5z=6$. [2]

Find the vector equation of the line passing through the point (2,3,2) and parallel to the line $\vec{r}=-2\hat{i}+3\hat{j}+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$

16a In any triangle ABC, using vector method, prove that $c=a\cos B+b\cos A$ [4]

OR

b If $\vec{a},\vec{b}$ and $\vec{c}$ are unit coplanar vectors then find the scalar triple product of $[2\vec{a}-\vec{b},2\vec{b}-\vec{c},2\vec{c}-\vec{a}]$

17a Find the equation of plane passing through the point (1, —1, —1) and perpendicular to each of the planes $x-2y-8z=0$ and $2x+5y-z=0$. [4]

OR

b Show that the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})-\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect. Find their points of intersection.

18 Sketch the curve $y^2=4(2-x)$. Calculate the area bounded by the curve and the lines x = 0 and y =o. [6]

SECTION C (20 Marks)

19a The fixed cost of a new product is Rs.35, 000 and the variable cost per unit is Rs.500. If the demand function is p(x) = 500 – 100 x, find the marginal revenue [2] [2]

b The equations of the two regression lines obtained in a correlation analysis are as follows 4x+6y=21 and 8x+2y=11. Find the correlation coefficient [2]

c Given the total cost function for x units of a commodity as $C(x)=\frac{x^3}{3}+x^2+-8x+5$, *find* the marginal cost function and average cost function [4]

20a For observations of pairs (x, y) of variables x and y, the following results are obtained.

$\sum x=125, \sum y=100, \sum x^2=1{,}650, \sum y^2=1{,}500, \sum xy=50$ and $n=25$.

Find the equation of the line of regression of x on y. Estimate the value of x, if y=5.

OR

b Internal and external assessments were conducted on a group of 10 students. The marks were obtained in the assessments

Roll No	1	2	3	4	5	6	7	8	9
Internal Assessment	45	62	68	32	45	38	47	68	42
External Assessment	39	48	65	32	30	35	48	77	30

Find the line of best fit . and estimate the internal assessment value when internal assessr value is 48

21a If the total cost function is given by $C = a + bx + cx^2$ *verify that* $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$ [4]

OR

b The total cost and demand function of an item are given by $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $p = 100 - x$ respectively.

Write the total revenue function and the profit function. Find the number of items when the profit will be maximum. Find the maximum profit also

22 A manufacturer makes two types of calculators. Deluxe sells for Rs 120 and standard sells for Rs 100. It costs Rs 90 to produce a deluxe and Rs 80 to produce a standard calculator. in one week , manufacturer can produce from 200to 300 deluxe calculators and from 100 to 250 standard calculators, but no more than 500 total calculators. How many of each type should be produced per week to maximize the profit [6]