MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen Minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The Question Paper has **11 printed pages and one blank page.**
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C.**
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks, two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks each** in **Section B and Section C.**
10. While attempting **Multiple Choice Questions in Sections A, B and C,** you are required to **write only ONE option as the answer**.
11. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
12. Mathematical tables and graph papers are provided.
13. The intended marks for questions or parts of questions are given in the brackets [].

Instruction to Supervising Examiner

1. Kindly read **aloud** the Instructions given above to all the candidates present in the examination hall.

SECTION A – 65 MARKS

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If A is a square matrix of order 3 and its determinant is $|A| = -3$, then the value of $|-4A|$ is: [1]

(a) 202

(b) 192

(c) −212

(d) −192

(ii) Consider the function 'f' given by $f(x) = \log x,\ x > 0$, then the function 'f' is: [1]

(a) differentiable and continuous at $x = 1$.

(b) differentiable but not continuous at $x = 1$.

(c) continuous but not differentiable at $x = 1$.

(d) neither differentiable nor continuous at $x = 1$.

(iii) If events A and B are mutually exclusive, such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{2}{3}$, then the value of $P(A \cup B)$ is: [1]

(a) $\frac{11}{15}$

(b) $\frac{3}{15}$

(c) $\frac{14}{15}$

(d) $\frac{13}{15}$

(iv) **Assertion:** $f(x) = \begin{cases} 1 + x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$ at $x = 2$ is not differentiable. [1]

Reason: A function is said to be differentiable at $x = a$ if Left hand derivative is equal to Right hand derivative i.e., $Lf'(a) = Rf'(a)$.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(v) The value of $\int_0^{3/2}|x|\,dx$ is: **[1]**

(a) $\frac{1}{8}$

(b) $\frac{9}{8}$

(c) $\frac{9}{4}$

(d) $\frac{3}{4}$

(vi) **Statement 1:** If $0 < x < \frac{\pi}{2}$ then the value of $\tan^{-1}(\cot x) = \frac{\pi}{2} - x$ **[1]**

Statement 2: $\tan^{-1}(\tan x) = x, \forall x \in R$

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(vii) How many possible matrices can be formed of order 3×3 if each entry is either 0 or 1? **[1]**

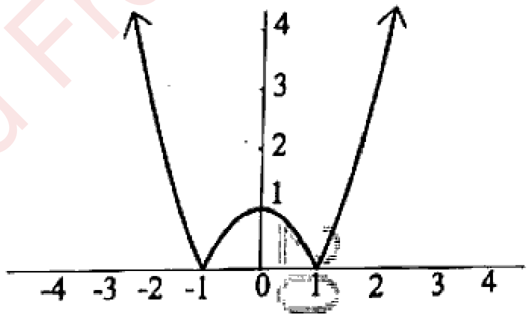
(a) 64

(b) 256

(c) 512

(d) 216

(viii) Observe the graph given below and answer the question that follows. **[1]**



Statement 1: $f(x)$ increases in $(-\infty, -1)$ and $(1, \infty)$

Statement 2: $f(x)$ decreases in $(-\infty, 0)$ and $(1, \infty)$

Which one of the following is correct?

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(ix) If set A contains four elements and set B contains five elements, then the number of one-one and onto mapping from $A \to B$ is: [1]

(a) 120

(b) 0

(c) 720

(d) 20

(x) The solution of $\frac{dy}{dx} - y = 1$, $y(0) = 1$ is given by: [1]

(a) $y = -e^{x} + 1$

(b) $y = -e^{-x-1}$

(c) $y = -1 + e^{x}$

(d) $y = 2e^{x} - 1$

(xi) **Assertion:** The system of three linear equations in three unknown variables can be written in the matrix form as AX = B. It has a unique solution $X = A^{-1}B$. [1]

Reason: Matrix A is non-singular.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xii) If $x = e^{y+e^{y+e^{y+\ldots\infty}}}$, $x > 0$ then find $\frac{dy}{dx}$. [1]

(xiii) Solve for x: $\begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix} = 86.$ [1]

(xiv) Find the principal value of $\sec^{-1}(-\sqrt{2})$. [1]

(xv) A relation R on the set A = {a, b, c} is defined by R = {(a, b), (b, a)}. Is the relation R symmetric? Justify. [1]

Question 2 [2]

Using properties of determinant, show that:

$$\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ 2(a-b) & 2(b-c) & 2(c-a) \end{vmatrix} = 0$$

Question 3 **[2]**

(i) Let $f(x) = 4 - (x-7)^3$ be an invertible function, then find $f^{-1}(x)$.

OR

(ii) Find the range of the function $f(x) = \frac{1}{3-2\sin x}$

Question 4 **[2]**

Evaluate: $\int \frac{e^x}{1+e^{2x}}\,dx$

Question 5 **[2]**

A die marked 1, 2, 3 in red and 4, 5, 6 in green is thrown. Let A be the event 'Number appearing is odd' and B be the event 'Number appearing is green'.

Prove that the events A and B are not independent.

Question 6 **[2]**

(i) The surface of a spherical balloon is increasing at the rate of 4 cm^2/sec. Find the rate of change of volume when its radius is 12 cm.

OR

(ii) Find the equation of the normal at (1, 2) to the curve $x^2 = 4y$.

Question 7 **[4]**

Vinayak runs a bakery shop. He sells three items: Sandwiches (₹x per unit), Fruit juices (₹y per unit) and Cookies (₹z per unit). The sales revenue over three days are 37, 26 and 37 respectively. The entire information is given below as matrix equation.

	Sandwiches	Fruit Juices	Cookies
D1	2	3	1
D2	1	2	3
D3	3	1	1

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 37 \\ 26 \\ 37 \end{pmatrix}$$

Consider $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ and $|A| = 17$. Find the price per unit for each item using matrix method.

Question 8 **[4]**

(i) If $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) = \alpha$, prove that $\sin 2\alpha = x^2$

OR

(ii) Solve for x: $2\tan^{-1}\left(\frac{1}{3}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) = \tan^{-1}x$

Question 9 **[4]**

If $y = x^3\log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$.

Question 10 **[4]**

(i) A sports store owner conducts a game 'weekend-surprise' every Friday for his customers.

He fills two bags with cricket balls of red and white colours. The first bag has 4 white and 4 red balls while the second bag contains 3 white and 5 red balls.

The rules of the game are:

- The customer will be blind-folded.
- Two balls have to be transferred from the first bag to the second bag one after another without replacement, and then one ball has to be drawn out from the second bag.
- The colours of the three balls (two balls transferred from the first bag and one ball drawn from the second bag) are considered.
- If all the three balls are of the same colour, the customer wins a surprise gift.

What is the probability that a customer can win the surprise gift?

OR

(ii) Children of a society practise building human pyramids for 16 days to participate in the pyramid building competition during the Janmashtami festival.

During practice sessions, the number of pyramids successfully formed in a day are X = 0, 1, 2, 3, 4. The data of the practice sessions is given in the following table:

Pyramids made (X)	0	1	2	3	4
No. of days	1	4	6	x	1

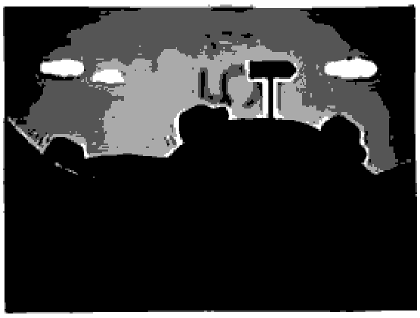
(a) Find the number of days on which the children made 3 pyramids. **[1]**

(b) Form a probability distribution table for the number of pyramids made per day. Verify if it is a valid probability distribution table. **[2]**

(c) Calculate the average number of pyramids formed. **[1]**

Question 11

A van is carrying a large amount of money in cash to deposit it in two ATM machines on a hill station. The location of these machines is at the turning points of the path traced by the van, given by the equation $h(x) = 2x^3 - 18x^2 + 48x + 3, (x \geq 0)$ where $h(x)$ is the height of the hill (in 100 m) at any point x.



(i) Prove that the van is at the height of 300 m when it starts moving. [1]

(ii) Find the location of the two ATM machines. [2]

(iii) Calculate the difference between the heights of the location of the two ATM machines. [1]

(iv) If the difference in the height of the location of the two ATM machines is greater than 1 km, then an extra armed security guard will be required. [1]
Based on the difference calculated in subpart (iii), determine if an extra armed guard will be required to protect the van.

(v) Find the absolute maxima and absolute minima for $h(x)$ in [0,4]. [1]

Question 12 [6]

(i) Evaluate: $\int \frac{\sin x \, dx}{\cos x(1-\sin x)}$

OR

(ii) Using properties of definite integral, calculate the value of: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin x \cos x} dx$

Question 13

A gardener wants to plant saplings on a day when rain is not predicted.
According to the forecast by the weather department,

- the probability of rain today is 0·4.
- if it rains today, the probability of it raining tomorrow is 0·8.
- if it does not rain today, the probability that it will rain tomorrow is 0·7.

(i) What is the probability that he will not plant the saplings tomorrow? [2]

(ii) Find the probability that he will plant them tomorrow. [1]

(iii) Given that he does not plant them tomorrow, what is the probability that he did not plant them today? [2]

(iv) What is the probability that he can plant saplings on both the days? [1]

Question 14

(i) Solve the following differential equation: [6]

$x^2 dy + (xy + y^2)dx = 0$

OR

(ii) Find the particular solution for the following differential equation:

$\sqrt{1 - y^2}\, dx = (\sin^{-1} y - x)dy$, given that $y(0) = 0$

SECTION B - 15 MARKS

Question 15

In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed.

(i) A scalar is multiplied by a unit vector, then the resultant is: [1]

Statement 1: A vector with the magnitude of the scalar.

Statement 2: A vector with unit magnitude.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(ii) The projection of $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ on $2\hat{\imath} - \hat{\jmath} + \hat{k}$ is: [1]

(a) $\frac{\sqrt{3}}{\sqrt{2}}$

(b) $\frac{-\sqrt{3}}{\sqrt{2}}$

(c) $\frac{-3}{2}$

(d) $\frac{-\sqrt{3}}{2}$

(iii) The direction cosines of the line passing through the points P(2, 3, 5) and Q(−1, 2, 4) are: [1]

(a) $\left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$

(b) $\left(\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$

(c) $\left(\frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$

(d) $\left(\frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$

(iv) Find the area of a parallelogram whose adjacent sides are given by the vectors: **[1]**
$\vec{a} = \vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} - 7\vec{j} + 4\vec{k}$

(v) Find the equation of the plane with intercepts 3, −4 and 2 on x, y and z axes respectively. **[1]**

Question 16 **[2]**

(i) Three drone cameras A, B and C are recording a hockey match. Their positions with respect to a control tower O are given by the following coordinates: A(1,4,6), B(3,4,5) and C(5,4,4).

Using vector method, show that the drone cameras A, B and C are moving in a straight path while recording the match.

OR

(ii) If $\vec{a}$ and $\vec{b}$ are mutually perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ *and* $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

Question 17

(i) The paths traced by two hot air balloons are: **[4]**

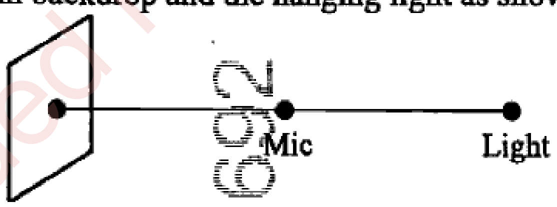
$$\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$$

Find the value of 'b' to be avoided so that the two hot air balloons do not collide.

OR

(ii) A school is preparing the stage for its annual day function. They want to place a hanging mic and a hanging light on the stage.

- They decide to position the mic at the point (3,2,1) such that it is equidistant from a plain backdrop and the hanging light as shown below.



- The equation of the surface of the plain backdrop is $2x - y + z + 1 = 0$

(a) Find the distance between the mic and the plain backdrop. **[1]**

(b) Calculate the coordinates of the position of the hanging light. **[3]**

Question 18 **[4]**

Find the area of the region bounded by $y = \sqrt{4 - x^2}$ and x axis using integration.

SECTION C – 15 MARKS

Question 19

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions.

(i) **Statement 1:** Two regression coefficients cannot have the same sign. [1]
Statement 2: Both the regression coefficients can be numerically greater than unity.

Which one of the following is correct?

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(ii) Which one of the following statements is true about Marginal Revenue? [1]

(a) It is always constant for all firms.

(b) It is always equal to the average revenue.

(c) It is the revenue gained from decreasing output by 1 unit.

(d) MR at $x = a$ is the additional revenue obtained by increasing the output from a to $a + 1$.

(iii) The cost of manufacturing x units of a commodity is $27 + 12x + 3x^2$. Find the output for which Average Cost is decreasing. [1]

(iv) For two variables x and y, if $\sigma_x = 5, r = \frac{-1}{2}, b_{yx} = \frac{-2}{7}$ then find the value of σ_y. [1]

(v) The total cost function for production and marketing of a product is given by $C(x) = \frac{3x^2}{4} - 7x + 3$, where x is the number of units produced. [1]
Find the level of output (number of units produced) for which MC = AC.

Question 20

(i) A company produces a commodity with ₹36,000 as a fixed cost. The variable cost is estimated to be 25% of the total revenue earned. The selling price of the product is ₹20 per unit.

Find the following:

(a) Cost function [1]

(b) Profit function [1]

OR

(ii) A school is organising an art and craft exhibition. The management has decided to donate the profit earned from the sale of exhibition items to an NGO. [2]

- Total cost function for organising the exhibition is:

$$C(x) = -x^2 + 11x + 50$$

- Each item is sold for ₹6.

Find the condition for the number of items to be sold to earn profit.

Question 21

(i) Consider the following data of a bivariate distribution:

- The mean of the variables x and y are 25 and 30 respectively.
- The regression coefficient of x on y is 0·4 and the regression coefficient of y on x is 1·6.

(a) Find the lines of best fit for the bivariate distribution. [2]

(b) Estimate the value of y when $x = 60$. [1]

(c) What is the coefficient of correlation between x and y? [1]

OR

(ii) If the regression lines of a bivariate distribution are $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$, then

(a) Calculate the arithmetic mean of x and y. [1]

(b) Estimate the value of x when $y = 7$. [2]

(c) Find the variance of y when $\sigma_x = 3$. [1]

Question 22 [4]

Raunak is a small-scale entrepreneur who sells sewing machines in a rural market. He wants to expand his business but has two main constraints: Capital and Storage.

He has a total capital of ₹5,760 to invest. The godown can store a maximum number of 20 sewing machines.

Raunak sells two types of machines:

- Electronic sewing machine, each costs him ₹360.
- Manually operated sewing machine, each costs him ₹240.

His wife Radhika suggests selling an electronic machine at a profit of ₹22 and a manually operated sewing machine at a profit of ₹18.

Using the concept of Linear Programming Problem, find the number of sewing machines of each type that Raunak should sell to maximise his profit.