S1003496

MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The Question Paper has **11 printed pages** and **one blank page**.
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C.**
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks**, **two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C.**
10. While attempting **Multiple Choice Questions in Section A, B and C**, you are required to **write only ONE option as the answer.**
11. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
12. Mathematical tables and graph papers are provided.
13. The intended marks for questions or parts of questions are given in the brackets [].

Instruction to Supervising Examiner

1. Kindly read aloud the instructions given above to all the candidates present in the examination hall.

SECTION A - 65 MARKS

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) Find the value of μ if the following system of equations is consistent. [1]
$2x + 3y - 8 = 0$, $7x - 5y + 3 = 0$, $4x - 6y + \mu = 0$

(a) 4

(b) 6

(c) 8

(d) 2

(ii) If $x = acos\theta, y = asin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ will be: [1]

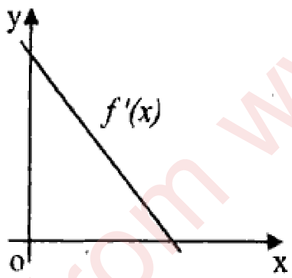
(a) -1

(b) Not defined

(c) $\frac{1}{a}$

(d) 0

(iii) Consider the graph of derivative of *f(x)* shown below. [1]



Assertion: *f(x)* is a quadratic function.

Reason: Derivative of every quadratic function is linear.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(iv) $\int \frac{\cos(logx)}{x} dx$ is equal to: [1]

(a) $log(sinx) + C$

(b) $cos(logx) + C$

(c) $logx + C$

(d) $sin(logx) + C$

(v) **Assertion:** If A is a skew-symmetric matrix of order 3×3, then det(A) = 0. [1]
Reason: If A is a square matrix, then det(A) = det(A')

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(vi) The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax$ is: [1]

(a) $\sqrt{1 - x^2}$

(b) $\frac{-1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}}$

(d) $-\sqrt{1 - x^2}$

(vii) **Statement 1:** Every scalar matrix is a diagonal matrix. [1]

Statement 2: Every diagonal matrix is an identity matrix.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(viii) The derivative of $\tan^{-1}\left(\frac{cosx-sinx}{cosx+sinx}\right)$ with respect to x, where $x \in \left(0, \frac{\pi}{2}\right)$ is: [1]

(a) $\frac{\pi}{4} + 1$

(b) 1

(c) $\frac{\pi}{4}$

(d) -1

(ix) If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$ then the value of $P(A|B) + P(B|A)$ is: [1]

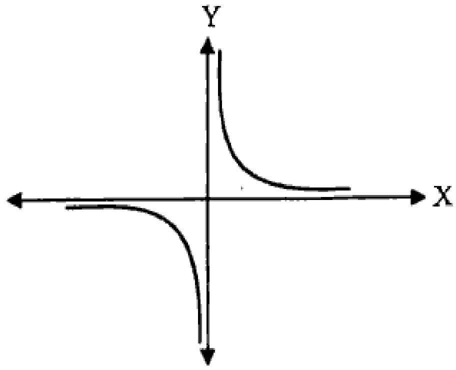
(a) $\frac{1}{12}$

(b) $\frac{7}{12}$

(c) $\frac{1}{10}$

(d) $\frac{1}{3}$

(x) Consider the following graph $f(x)$, $f: R \rightarrow R$ [1]



Statement 1: The function is one-one function.

Statement 2: The function is onto function.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(xi) $2\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} + \begin{vmatrix} x & y & z \\ s & t & u \\ 2a & 2b & 2c \end{vmatrix}$ [1]

Which one of the following is equal to the above sum?

(a) $\begin{vmatrix} 3x & 3y & 3z \\ 2p+s & 2q+t & 2rt+u \\ 4a & 4b & 4c \end{vmatrix}$

(b) $\begin{vmatrix} 2x & 2y & 2z \\ 2p+s & 2q+t & 2r+u \\ 2a & 2b & 2c \end{vmatrix}$

(c) $\begin{vmatrix} x & y & z \\ p+s & q+t & r+u \\ 2a & 2b & 2c \end{vmatrix}$

(d) $\begin{vmatrix} 2x & 2y & 2z \\ p+s & q+t & r+u \\ 4a & 4b & 4c \end{vmatrix}$

(xii) In a class of 50 students, 25 study English, 10 study History and 10 study both the subjects. Find the probability that a randomly selected student studies either English or History. [1]

(xiii) If R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$, find the range of R^{-1}. [1]

(xiv) For what value of a, is the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ a & 2 & -3 \end{pmatrix}$ not invertible? [1]

(xv) Consider the two events A and B. The probability that at least one of A or B occurs is 0·6. If the probability of both A and B occurring is 0·3, then calculate $P(A') + P(B')$. [1]

Question 2 [2]

Evaluate $\int_{-2}^{2} x f(x)\, dx$ given, $f(x) = x + g(x)$ where $g(x)$ is an even function.

Question 3 [2]

(i) If $\int x^{-3}\, 5^{\frac{1}{x^2}}\, dx = K\, 5^{\frac{1}{x^2}} + C$, then find the value of K.

OR

(ii) Evaluate: $\int \left(\frac{x^2 + \sin^2 x}{1 + x^2}\right) \sec^2 x\, dx$

Question 4 [2]

Find the equation of the normal to the curve $y = x^2 - 3x + 1$ at the point (3, 1).

Question 5 [2]

(i) Evaluate: $\int_0^{\frac{\pi}{2}} \log \tan x\, dx$

OR

(ii) Solve the equation: $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Question 6 [2]

In a market survey, three commodities A, B and C were studied. Each commodity had three varieties. To find out the index number of A, B and C fixed weights were assigned to the three varieties of each of the commodities.

The table given below shows the result of the survey.

Commodity variety	Variety			Total weight
	I	II	III	
A	1	2	3	P
B	2	4	5	Q
C	3	5	6	R

Answer the following questions:

(i) Represent the above information in a matrix form.

(ii) Find the value of P, Q and R if the weights assigned to A, B and C are 2 kg, 3 kg and 1 kg respectively.

Question 7 **[4]**

Using the properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Question 8 **[4]**

(i) If $x = \sin t, y = \sin pt$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

OR

(ii) In a particular forest, the population density (P) of wild animals (in thousands of animals per sq. km.) at a distance 'r' km from the centre of the forest is approximately given by $P = 10 + 20r - 10r^2$

(a) What is the population density in the centre of the forest?

(b) Calculate the rate of change of population density.

(c) For what value of 'r' is the population density maximum?

(d) Prove that the population density of animals varies with distance from the centre of the forest.

Question 9 **[4]**

(i) Evaluate: $\int x \tan^{-1} x \, dx$

OR

(ii) Evaluate: $\int \frac{\cos x \, dx}{\sin^2 x + 4\sin x + 5}$

Question 10 **[4]**

In a school, there are 30 teachers in the examination committee. Out of these, 20 never commit any error in their work. Two teachers are selected at random from the committee. The random variable X is the number of selected teachers who never make an error in their work.

(i) What are the possible values that X can take?

(ii) Find the probability distribution of this random variable.

(iii) Find the mean of this distribution.

Question 11 [6]

(i) Evaluate: $\int_0^{2\pi} \frac{x\cot x}{\operatorname{cosec}x+\cot x}\,dx\,,\, x \neq \pi$

OR

(ii) Evaluate: $\int \frac{dx}{x^3+1}$

Question 12 [6]

(i) Solve the differential equation: $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$. Find the particular solution satisfying the condition $y = 0$ when $x = 0$.

OR

(ii) Find the particular solution of the differential equation:
$(x+y)dy + (x-y)dx = 0$, given that $y = 1$ when $x = 1$

Question 13 [6]

Naman has to attend his friend's marriage. He can reach the wedding venue by train, bus or a two-wheeler. The probability of Naman using these as means of transport are $\frac{3}{10}, \frac{1}{5}$ and $\frac{1}{2}$ respectively.

The probability of Naman reaching the wedding venue on time are $\frac{3}{4}, \frac{2}{3}$ and $\frac{11}{12}$ if he uses train, bus and two-wheeler respectively. Naman reached the venue late.

What is the probability that he travelled by train?

Question 14

Consider the function $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

(i) Find the range of y if $x = 0, \frac{1}{2}$ and $\frac{-1}{2}$ [1]

(ii) Prove: $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ [1]

(iii) Evaluate: $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$ [2]

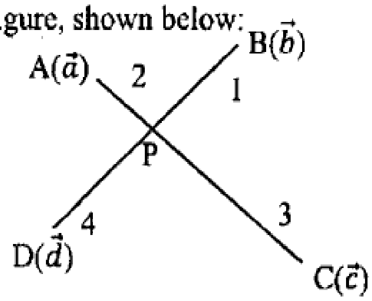
(iv) If $4\sin^{-1}x + \cos^{-1}x = \pi$, calculate the value of 'x'. [2]

SECTION B - 15 MARKS

Question 15

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Consider the figure, shown below: [1]



Statement 1: The position vector of P is $\frac{2\vec{c}+3\vec{a}}{5}$

Statement 2: The position vector of P is $\frac{4\vec{d}+\vec{b}}{5}$

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(ii) There are two solar panels, A and B placed parallel to each other on the terrace of a building. The equation of the surface of the panel A is $2x + 2y - z + 9 = 0$

(a) The direction cosines of the plane are: [1]

(1) $\left(\frac{-2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

(2) $\left(\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$

(3) $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(4) $\left(\frac{-2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$

(b) A fly is sitting on panel A. The position of the fly is $(2, m, 3)$. Find the value of 'm'. [1]

(c) Find the equation of the panel B if the point $(3, 2, -4)$ lies on it. [1]

(d) Find the distance between the panels. [1]

Question 16

[2]

(i) Find the shortest distance between the lines:

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda\,(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

$\vec{r} = (2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu\,(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$

OR

(ii) Find the equation of the straight line passing through $(1, 2, 1)$ and parallel to the line joining the points $(1, 4, 6)$ and $(5, 4, 4)$.

Question 17

Consider the two vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 5\hat{\imath} + q\hat{\jmath} - \hat{k}$.

(i) Calculate 'q' if both the vectors are perpendicular to each other. [1]

(ii) Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ if $q = 4$ [2]

(iii) Find the projection of $\vec{a}$ on $\vec{b}$ if $q = 1$. [1]

Question 18

[4]

(i) Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

OR

(ii) Find the area enclosed by the parabola $y^2 = x$ and the line $y = 2x$.

SECTION C - 15 MARKS

Question 19

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Which one of the following statements is correct? [1]

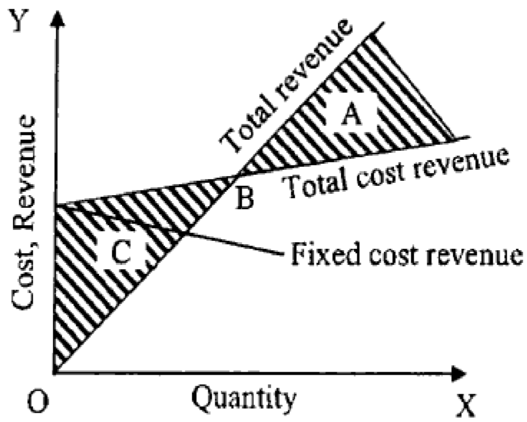
(a) The regression coefficient is the geometric mean of the correlation coefficient.

(b) The correlation coefficient is the arithmetic mean between the regression coefficients.

(c) The correlation coefficient is the geometric mean between the regression coefficients.

(d) The correlation coefficient is the harmonic mean between the regression coefficients.

(ii) Observe the graph shown below of Cost, Revenue versus Quantity. [1]



Statement 1: A denotes Profit area, B denotes Break-even point and C shows Loss area.

Statement 2: A denotes Loss area, B denotes Average of total revenue and total cost and C shows Profit area.

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(iii) Consider the following table:

x	3	5	?	6
y	1	?	3	2

(a) Complete the table if $\bar{x} = 4, \ \bar{y} = 2$ [1]

(b) If $b_{xy} = \frac{-1}{2}$ and $r^2 = \frac{1}{20}$ calculate b_{yx} [1]

(c) Frame the line of best fit y on x. [1]

Question 20 [2]

(i) Consider the data given below:

	x	y
Arithmetic mean	36	85
Standard deviation	11	8

and $r = 0{\cdot}66$

Find the regression coefficients.

OR

(ii) Out of the two regression lines given below, find the line of regression of x on y.

$x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$

Question 21 [4]

(i) The cost function $C(x)$ of producing 'x' quantities of a product is given by $C(x) = 500x^2 + 2500x + 5000$. If each unit of the product is sold at ₹ 6000 then, calculate

(a) Break-even point

(b) Marginal cost and show that the marginal cost increases as the output 'x' increases.

OR

(ii) The demand function for a product is given by

$$p = 15900 - 9x - 2x^2$$

(a) Find the revenue function.

(b) Write the marginal revenue.

(c) Calculate the level of output at which the total revenue is maximum.

Question 22 [4]

A cooperative society of farmers has 50 hectares of land to grow two crops, X and Y. The profits from crops, X and Y per hectare are estimated as ₹ 10500 and ₹ 9000 respectively. To control weeds, a liquid herbicide has to be used on crops X and Y in the quantity of 20 litres and 10 litres per hectare respectively.
Further, the quantity of herbicide to be used on both the crops should not exceed 800 litres to protect the environment.
How much land should be allocated to each crop to maximise the total profit of the society?
Formulate the above Linear Programming Problem mathematically and then solve it graphically.