20a. For observations of pairs $(x, y)$ of variables x and y, the following results are obtained. [4]

$\sum x = 125, \sum y = 100, \sum x^2 = 1{,}650, \sum y^2 = 1{,}500, \sum xy = 50$ and $n = 25$.

Find the equation of the line of regression of x ony. Estimate the value of x, if $y = 5$.

OR

Internal and external assessments were conducted on a group of 10 students. The marks were obtained in the assessments

| Roll No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Internal Assessment | 45 | 62 | 68 | 32 | 45 | 38 | 47 | 68 | 42 |
| External Assessment | 39 | 48 | 65 | 32 | 30 | 35 | 48 | 77 | 30 |

Find the line of best fit . and estimate the internal assessment value when internal assessr value is 48

21a. If the total cost function is given by $C = a + bx + cx^2$ verify that $\frac{d\,(AC)}{dx} = \frac{1}{x}(MC - AC)$ [4]

22. The total cost and demand function of an item are given by $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $p = 100 - x$ respectively. Write the total revenue function and the profit function. Find the number of items when the profit will be maximum. Find the maximum profit also.



# Half Yearly Examination 2018-2019

## MATHEMATICS

**Class : XII**

**Time : 3 Hrs.** **Full Marks : 100**

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.) The Question Paper consists of **three** sections A, B and C. Candidate are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C**

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer. The intended marks for questions or parts of questions are given in brackets. [ ]

### Section A

1. a) If the binary operation * on I is defined as a*b=3a+4b-8, find the Identity element, and inverse of 5 [2]

b) Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ [2]

c) Without expanding find the value of [2]

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

d) Evaluate $\int_0^4 |2x - 1|\,dx$ [2]

e) Find $\frac{d^2y}{dx^2}$ if $x = a(1 + \sin t)$ and $y = a\cos t$ [2]

f) If $[2x \;\; 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$ find the values of x [2]

{Turn Over}

g) Evaluate $\lim_{x \to 0} \frac{x^3 \cot x}{1 - \cos x}$ [2]

h) A card is drawn from a well shuffled pack of playing cards. What is the probability that it is neither a hearts nor a Queen? [2]

i) A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the probability that the no.2 has appeared at least once. [2]

j) Show that $\sin^{-1}\frac{\sqrt{3}}{2} + 2\tan^{-1}\frac{1}{\sqrt{3}} = \frac{2\pi}{3}$ [2]

2) Show that the function F;W-w defined by [4]

$F(X) = \begin{cases} N+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$, is bijective.

3a. Show that

$$\begin{vmatrix} 1 & a^2 + bc & a^3 \\ 1 & b^2 + ca & b^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

4. Solve for x, $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x-2)$ [4]

5a Is this function defined by $f(x) = \begin{cases} \frac{x}{\sin 2x} & \text{when } x = 0 \\ 2, & \text{when } x = 0 \end{cases}$ continuous at x=0

OR

b Using Langrage's mean value theorem, find a point on the curve $y = (x - 3)^2$ defined in the interval [3, 4] where the tangent is parallel to the chord joining the end point of the curve [4]



## SECTION B (20 Marks)

15a. Find the area of the parallelogram whose adjacent sides are given by the vectors $2\vec{i} - \vec{j} + 2\vec{k}$ and $3\vec{i} - 2\vec{j} + \vec{k}$ [2]

b. Find the angle between the planes $2x + 3y + 4z = 5$ and $3x - 4y - 3z = 6$. [2]

c. Find the vector equation of the line passing through the point (2,3,2) and parallel to the line $\vec{r} = -2\hat{i} + 3\hat{j} + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

16a. In any triangle *ABC*, using vector method, prove that $c = a\cos B + b\cos A$ [4]

OR

b. If a, b and c are unit coplanar vectors then find the scalar triple product of $[2\vec{a} - \vec{b}, 2\vec{b} - \vec{c}, 2\vec{c} - \vec{a}]$ [4]

17 Find the equation of plane passing through the point (1, −1, −1) and perpendicular to each of the planes $x - 2y - 8z = 0$ and $2x + 5y - z = 0$.

18. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) - \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Find their points of intersection. [6]

## SECTION C (20 Marks)

19a. Find the profit function and break-even points when $C(x) = 300x^2 + 4200x + 13500$ each product is sold for Rs 8400 [2]

b. The equations of the two regression lines obtained in a correlation analysis are as follows 4x+6y=21 and 8x+2y=ll. Find the correlation coefficient [2]

c. Given the total cost function for x units of a commodity as $C(x) = \frac{x^3}{3} + 5x^2 - x + 25$ find the marginal cost function and average cost function. [4]

*{Turn Over}*

6. If $y = (\log(x+\sqrt{x^2 + a^2}))^2$, show that [4]

$$(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$$

7a. 1) $\int \frac{4}{(2\sin x+3\cos x)^2}\,dx$ [4]

OR

b. Evaluate as a limit of a sum $\int_0^4 (x^2 + 2x + 1)\,dx$ [4]

8a. Water is leaking from a conical funnel at the rate of $5cm^3$/sec. if the radius of the base of the funnel is 10 cm and its height is 20cm, find the rate at which the water level is dropping when it is 5 cm from the top.

OR

b. Find the equation of the normal to the curve $3x^2 - y^2 = 2$ which are parallel to line $x+3y=2$. [4]

$$\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 9)}\,dx$$

10. A box has 20 pens of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement at most 2 are defective. [4]

lla. Show that that following equations are consistent $x - 2y + z = 0$, $y-z = 2$, $2x-3z = 10$. [6]
Also find the solution using matrix method.

OR

b. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 10 \\ 9 & 8 & 7 \end{bmatrix}$$

{Turn Over}



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{Turn Over}

3

12a. If the sum of the lengths of the hypotonus and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$ [6]

OR

b. Show that the volume of the largest circular cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.

13. Evaluate $\int \frac{dx}{(x^4 + 1)}$ [6]

14a. Find Let X denote the number of hours you study during randomly selected school day. [6]

The probability that can take the values "x" has the following form, where 'k' is some unknown constant.

$$P(X = x) = \begin{cases} 0.1 \ if \ x = 0 \\ kx \ if \ x = 1 \ or \ 2 \\ k(5 - x) \ if \ x = 3or4 \\ 0 \ otherwise \end{cases}$$

the value of k find the mean standard deviation of hours of study

OR

A doctor visit to a patient. From past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are $\frac{3}{10}$ $\frac{1}{5}$ $\frac{1}{10}$ , *and* $\frac{2}{5}$ respectively. The probability that he will be late are $\frac{1}{4}$ , $\frac{1}{3}$ , $\frac{1}{12}$ if e come by other means of transport., then he will not be late. When he arrives, he is late. What is the probability that he comes by train.

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4